

Program of the PhD lecture course

Prof. Thomas Schick (Universität Göttingen)

Title: *The topology of positive scalar curvature*

16 hours

Description.

When does a differentiable manifold admit a metric of positive scalar curvature? Already for closed compact surfaces one sees, from the Gauss-Bonnet theorem, that this question is intimately related to the topology of the manifold. And if the space of metric of positive scalar curvature is non empty, what is its topology? For example: what are its homotopy groups?

Answering to these questions (for spin manifolds) involves several different techniques:

- elliptic theory, K-Theory and C^* -algebras, in order to find obstructions to the existence of positive scalar curvature and connect them to the topology of the manifold (Gromov, Lawson, Rosenberg);
- the theory of minimal hypersurfaces, again to find obstructions to the existence of metric of positive scalar curvature (Schoen, Yau);
- the study of bordism groups in order to find sufficient conditions for the existence (Stolz);
- the investigation of the topology of diffeomorphism groups and surgery theory in order to investigate the topology of the space of positive scalar curvature (Schick, Botvinnik, Randall-Williams).

The goal of this course is to illustrate these techniques and introduce students to this fascinating and very active area of research.

Syllabus.

- scalar curvature as a basic invariant in Riemannian Geometry
- scalar curvature and minimal hypersurfaces (following Schoen and Yau)
- the spin Dirac operator and scalar curvature: obstructions to existence of positive scalar curvature
- higher index theory via non-commutative geometry and K-theory: more obstructions to positive scalar curvature. The Gromov-Lawson-Rosenberg conjecture

- relative index theory: probing the space of metrics of positive scalar curvature
- constructions of positive scalar curvature metrics using the topology of the group of diffeomorphisms
- constructions of positive scalar curvature metrics as a surgeon
- open questions and further directions of research

(time is the probably up here; if not, more material will be added).