THE ASSESSMENT OF THE INTERVENTIONS ON TRANSPORTATION SYSTEMS: MULTI-ATTRIBUTE VALUE THEORY

Paolo Delle Site

Niccolò Cusano University Rome
Multi-criteria methods

- Used to assess interventions

<table>
<thead>
<tr>
<th>Method</th>
<th>Criteria</th>
<th>Results</th>
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</thead>
</table>
| Cost-benefit analysis       | Only tangible criteria which need to be monetised | - Convenience judgement on a single intervention  
|                             |                                               | - Preference ordering on a set of interventions                        |
| Multi-criteria methods      | Any criterion                                 | - Preference ordering on a set of interventions                        |
Multi-criteria methods: classification

<table>
<thead>
<tr>
<th>Compensatory methods</th>
<th>Outranking methods</th>
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<td>Multi-Attribute Value Theory (MAVT)</td>
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<td>Multi-Attribute Utility Theory (MAUT)</td>
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<td></td>
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</table>
Multi-attribute value theory

- $A = \{x_1, \ldots, x_m\}$ set of $m$ alternatives
- $n$ criteria
- One evaluator (policy maker or expert)
- A weight $w_j$ is assigned to each criterion such that
  
  \[ 0 < w_j < 1, \quad j = 1, \ldots, n \]
  
  \[ \sum_{j=1}^{n} w_j = 1 \]

- A rating or value $v_j(x_i)$ is assigned to the performance of the alternative with respect to each criterion $j$ by a value scale between 0 (worst performance) and 100 (best performance)

- The total rating or value of the alternative is computed:
  
  \[ v(x_i) = \sum_{j=1}^{n} w_j \cdot v_j(x_i) \quad i = 1, \ldots, m \]

- We have: $0 \leq v(x_i) \leq 100$
Value functions

LINEAR VALUE

DECREASING MARGINAL VALUE

Indicator for the criterion
Linear value function

a quantitative indicator $g_j(x_i)$ of the performance of the alternative with respect to the criterion is available:

- the rating is assigned

$$v_j(x_i) = \frac{g_j(x_i) - g_j(x_j^-)}{g_j(x_j^+) - g_j(x_j^-)} \cdot 100$$

$$v_j(x_j^-) = 0$$

$$v_j(x_j^+) = 100$$

$x_j^-$ alternative, among those to be evaluated, with the worst performance with respect to criterion $j$

$x_j^+$ alternative, among those to be evaluated, with the best performance with respect to criterion $j$
Example with investment costs

\[ C = 50, 100, 200, 500 \]
\[ \nu(C = 500) = 0 \]
\[ \nu(C = 50) = 100 \]
\[ \frac{100 - 500}{50 - 500} = \frac{-400}{-450} = 88.9 \]
\[ \frac{200 - 500}{50 - 500} = \frac{-300}{-450} = 66.7 \]
a quantitative indicator $g_j(x_i)$ of the performance of the alternative with respect to the criterion is available:

- **DIRECT RATING** or **DIRECT SCORING**: direct numerical rating from 0 to 100

- **DIRECT MIDPOINT RATING**

\[
\begin{align*}
  v_j(x_j^-) &= 0 \\
  v_j(x_j^+) &= 100
\end{align*}
\]

the evaluator is asked which level $z_j$ is midway in value between $x_j^-$ and $x_j^+$

\[
v_j(z_j) = \frac{1}{2} \cdot [v_j(x_j^-) + v_j(x_j^+)]
\]

the question is repeated
only a descriptor (narrative) of the indicator associated with the criterion is available (finite number of levels):

- DIRECT RATING or DIRECT SCORING: direct numerical rating from 0 to 100 assigned to each level
Weight estimation

- Given the 2 fictitious alternatives:
  - $x_j^1$ with the worst performance with respect to criterion $j$, and with worst values of performance with respect to the other criteria
  - $x_j^2$ with the best performance with respect to criterion $j$, and with the same values as alternative 1 of the performance with respect to the other criteria
- **Swing**:
  - $v(x_j^2) - v(x_j^1) = w_j \cdot v_j(x_j^+) - w_j \cdot v_j(x_j^-) = w_j \cdot 100 - w_j \cdot 0 = 100 \cdot w_j$
Direct method: ratio weighting

- The criterion with the highest swing is identified, based on the subjective judgment of the expert or of the policy maker. A rating 100 is assigned to this criterion.
- The swings of the other criteria are evaluated as percentage $p_j$ of the highest value swing.
- Once the ratings have been assigned to criteria based on the respective swings, weights are given by:

$$w_j = \frac{p_j}{\sum_{k=1}^{n} p_k} \cdot \frac{1}{100}$$
“Given an alternative with the worst outcome for each attribute, which attribute would you change first from worst to best? Which second? ... 
Assign 100 to the first. Assign between 0 and 100 to the second. ...”
### Example: layout alternatives of a metro line

<table>
<thead>
<tr>
<th>attributes</th>
<th>layout alternative</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>base</td>
<td>complete</td>
<td></td>
</tr>
<tr>
<td>1. construction costs</td>
<td>1,7 billion €</td>
<td>2,4 billion €</td>
<td></td>
</tr>
<tr>
<td>2. travel time benefits</td>
<td>360,000 daily users of the new line have an average saving of 12 minutes per trip</td>
<td>500,000 daily users of the new line have an average saving of 12 minutes per trip</td>
<td></td>
</tr>
<tr>
<td>3. road safety benefits</td>
<td>in 1 year: 67 accidents, fatal and with injured, are saved, of them 3% are fatal</td>
<td>n 1 year: 101 accidents, fatal and with injured, are saved, of them 3% are fatal</td>
<td></td>
</tr>
<tr>
<td>4. benefits relating to the effects on citizens’ health of particulate matter emissions</td>
<td>in 1 year: 90 premature deaths due to cardio-respiratory diseases are saved, 4100 days where activity is restricted due to health conditions are saved</td>
<td>in 1 year: 158 premature deaths due to cardio-respiratory diseases are saved, 7200 days where activity is restricted due to health conditions are saved</td>
<td></td>
</tr>
</tbody>
</table>
Example: layout alternatives of a metro line (2)

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weights</th>
<th>Ratings on the criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rating for swing</td>
<td>Normalised weight</td>
</tr>
<tr>
<td>Costs</td>
<td>100</td>
<td>0.5</td>
</tr>
<tr>
<td>Benefits: travel times</td>
<td>33</td>
<td>0.167</td>
</tr>
<tr>
<td>Benefits: safety</td>
<td>33</td>
<td>0.167</td>
</tr>
<tr>
<td>Benefits: emissions</td>
<td>33</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>Total rating</td>
<td>50</td>
</tr>
</tbody>
</table>
Measuring Attractiveness by a Categorical Based Evaluation Technique (Carlos Bana e Costa and Jean-Claude Vansnick)

- *Ordinal preference* information: ranking of elements in decreasing order of attractiveness
- *Cardinal preference* information in the form of an interval scale: how much an element is more attractive than another element

Questions:
- comparative judgements
- judgements of difference of attractiveness
Estimation of value functions

- The evaluator is asked to verbally judge the difference of attractiveness between each two levels $x$ and $y$, with $x$ more attractive than $y$, choosing one of the following semantic categories:
  - $C_1$ very weak difference of attractiveness
  - $C_2$ weak difference of attractiveness
  - $C_3$ moderate difference of attractiveness
  - $C_4$ strong difference of attractiveness
  - $C_5$ very strong difference of attractiveness
  - $C_6$ extreme difference of attractiveness
MACBETH checks if there exists a numerical scale $\varphi$ that satisfies the following two conditions (measurement rules):

- **Condition 1**: ordinal condition
  $$\forall x, y: \varphi(x) > \varphi(y) \iff x \text{ is more attractive than } y$$

- **Condition 2**: semantic condition
  $$\forall k, k' \in \{1, 2, 3, 4, 5, 6\},$$
  $$\forall x, y, w, z \text{ with } (x, y) \in C_k, (w, z) \in C_{k'},$$
  $$k \geq k' + 1 \Rightarrow \varphi(x) - \varphi(y) > \varphi(w) - \varphi(z)$$
### Estimation of value functions (3)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>Current scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>no</td>
<td>weak</td>
<td>moderate</td>
<td>moderate</td>
<td>very</td>
<td>extreme</td>
<td>15.00</td>
</tr>
<tr>
<td>B</td>
<td>no</td>
<td>weak</td>
<td>weak</td>
<td>very</td>
<td>extreme</td>
<td>13.00</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>no</td>
<td>very weak</td>
<td>strong</td>
<td>very</td>
<td></td>
<td>11.00</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>no</td>
<td>strong</td>
<td>very</td>
<td></td>
<td></td>
<td>10.00</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>no</td>
<td>moderate</td>
<td></td>
<td></td>
<td></td>
<td>4.00</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>no</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
Dealing with incompatibility

1. INCOHERENCE: conflict between comparative and semantic judgements
Example:
A is more attractive than B: \( \varphi(A) > \varphi(B) \)
but difference of attractiveness B and D strong, A and D moderate:
\[
\varphi(B) - \varphi(D) > \varphi(A) - \varphi(D) \implies \varphi(B) > \varphi(A)
\]

2. SEMANTIC INCONSISTENCY: conflict between semantic judgements
Example:
Weight estimation

- Consider the fictitious alternatives:
  \[ x_0 = (\text{worst}_1, ..., \text{worst}_n) \]
  \[ x_i = (\text{worst}_1, \ldots \text{worst}_{i-1}, \text{best}_i, \text{worst}_{i+1}, \ldots \text{worst}_n) \]
  \[ i = 1, ..., n \]

- The evaluator is asked to judge the difference of attractiveness between each \( x_i, i = 1, ..., n \) and \( x_0 \) according to the semantic categories; this produces a ranking of \( w_i, i = 1, ..., n \)

- The evaluator is asked to judge the difference of attractiveness between \( x_i \) and \( x_j \) with \( w_i > w_j \) according to the semantic categories; this produces a matrix of pair judgements from which weights \( w_i, i = 1, ..., n \) are derived
Weight estimation (2)
Concordance analysis: ELECTRE I method

- $A = \{x_1, ..., x_m\}$ set of $m$ alternatives
- $n$ criteria
- Let $g_j$ be a function defined on $A$ such that the condition $g_j(x_i) > g_j(x_k)$ is equivalent to state that the alternative $x_i$ is better than alternative $x_k$ from the point of view of the criterion $j$.
- We assume the same measurement scale is used for all criteria.
- The alternative $x_i$ is dominant on alternative $x_k$ if and only if we have:

$$g_j(x_i) \geq g_j(x_k) \quad \forall j$$

with strict inequallity sign for at least one criterion.
To each criterion $j$ a weight positive and increasing with the importance is assigned such that

$$0 < w_j < 1, \quad j = 1, \ldots, n$$

$$\sum_{j=1}^{n} w_j = 1$$

Concordance index:

$$c_{ik} = \sum_{j \in \Phi_{ik}} w_j$$

where $\Phi_{ik}$ denotes the set of criteria for which $x_i$ is better or indifferent to $x_k$, that is $g_j(x_i) \geq g_j(x_k)$

Discordance index:

$$d_{ik} = \frac{1}{\delta} \cdot \max_j [g_j(x_k) - g_j(x_i)]$$

$$\delta = \max_{i,k,j} [g_j(x_k) - g_j(x_i)]$$
Concordance analysis: ELECTRE I method (3)

- We fix: a concordance threshold \( p \) sufficiently high, and a discordance threshold \( q \) sufficiently low, typically \( p > 0.5 \) e \( q < 0.5 \).

- Alternative \( x_i \) outranks \( x_k \) if and only if:

\[
\begin{cases}
    c_{ik} \geq p \\
    d_{ik} \leq q
\end{cases}
\]

- A subset \( N \) of the alternatives such that:

\[
\forall x_k \in A \setminus N \quad \exists x_i \in N: x_i S x_k \\
\forall x_i, x_k \in N \quad x_i \not< x_k \text{ and } x_k \not< x_i
\]
Result produced by the methodology

- N: “superior” alternatives;  A\N: “inferior“ alternatives
Summary of main topics

- Multi-criteria methods
  - difference with cost-benefit analysis
  - classification
- Multi-attribute value theory (MAVT)
  - direct methods
  - MACBETH
- Concordance analysis
  - ELECTRE I
References


Contacts

prof. ing. Paolo Delle Site

paolo.dellesite@unicusano.it