

PHD COURSE ON REAL ANALYSIS

Scuola di Scienze Statistiche
Sapienza Università di Roma

Programme:

Abstract integration and Borel measurability. Recalls on topology. σ -algebras, measurable spaces, measurable functions. Continuity and examples of measurable functions. Borel sets, Borel measurable functions. Simple functions, positive measures and first properties. Recalls on infinite arithmetics. Lebesgue integral. Monotone and dominated convergence theorem. Fatou Lemma. Remarks on null sets.

Lebesgue measure. Topological preliminaries: compact and locally compact spaces, vector spaces. Compact support functions. Riesz representation theorem. Lebesgue measure. Continuity of Lebesgue measurable functions: Lusin theorem and Vitali-Carathéodory theorem. Absolute continuity, Radon-Nicodým theorem.

L^p spaces. Convex functions. Jensen, Holder and Minkowski inequalities. L^p spaces. L^p norm and completeness of L^p . Approximation with continuous functions. Duality.

Product measures. Measurability on product sets. Fubini Theorem. Complete measures and completion of product measures. Convolutions.

Prerequisites: Interested students should be familiar with the theoretical aspects of the courses on Mathematical Analysis I and II, and Geometry. In particular, we rely on basic notions on topology and linear algebra: open, closed, compact sets and related set operations, infinite arithmetics, vector spaces.

Students are strongly encouraged to also review the following topics: definition of limit and continuity for real valued functions (of one or more real variables), limit of sequences, definition and construction of Riemann integral, examples of non-integrable functions (e.g., Dirichlet function) and basic theorems on integration (Fundamental Theorem of Calculus, mean value theorem for integrals, change of variables).

Textbook: Rudin, Walter. *Real and complex analysis*. Tata McGraw-hill education, 2006.

Italian edition: Rudin, Walter. *Analisi reale e complessa*. Bollati Boringhieri, 1996.

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Course website:

<https://sites.google.com/uniroma1.it/real-analysis/home-page>