PROBABILITY THEORY I: MEASURE THEORY
PHD IN STATISTICS – UNIVERSITY OF ROME LA SAPIENZA
ACADEMIC YEAR 2020-2021

Lecturer: Alessandro De Gregorio

Brief course description: The course will cover the main topics in Probability Theory (see the course topics below). For a complete and rigorous development of the matter, an approach based on the measure theory will be adopt.

Course topics:

- **Measure spaces:** Classes of sets. Borel σ-algebras. Definition of measure spaces. Probability measures and its properties. Spaces of probability. π-systems and Caratheodory's extension theorem. Lebesgue measure. Events.
- **Random variables:** Definition of random variable and measurability. Examples. σ-algebras generated by a collection of functions. Probability law and distribution function. Existence of a random variable and Skorohod representation. Monotone class theorem.
- **Independence:** Definition of independence: σ-algebras, random variables, events. Borel-Cantelli Lemma. Coin-tossing model. Tail σ-algebra and Kolmogorov's 0-1 law.
- **Expectation:** Definition of expectation. Integration with respect to a probability measure. L^p-norms. Convergence theorems. Inequalities: Markov, Cauchy-Schwarz, Jensen. Expectation rule.
- **Product measure:** Product measure. Fubini's theorem. Joint laws. Independence and product measures. Infinite products of probability triples.
- **Weak convergence:** Definition of weak convergence. Helly-Bray Lemma. Tightness. Central limit theorem.

Textbooks:

- Williams D. *Probability with Martingales*. Cambridge University Press (1991).
- Billinglsey, P. *Probability and Measure*. John Wiley & Sons (1995).

MARTINGALE THEORY

Programme of the course:

- Conditional expectation
- Filtered spaces and adapted processes
- Definition of Martingales and examples
- Previsible processes
- Stopping times and Doob's optional stopping theorem
- Martingales bounded in L^2: orthogonality of increments
- Kolmogorov's Strong Law of Large Numbers
- Doob decomposition
- Applications: option prising, discrete Black-Scholes formula.

Textbook:
David Williams. Probability with Martingales. Cambridge University Press 2018.

Calendario (da definire): 6 lezioni di 2 ore.

LEVY PROCESSES

L.Beghin

PROGRAMME

- Characteristic function and its properties.
- Infinite divisibility.
- Examples of infinitely divisible laws.
- Lévy-Khinchine characterization.
- Stable laws: definitions and examples.
- Lévy processes: theory and applications.
- Subordinators and time-change.
- Stochastic processes linked to fractional equations.

TEXTBOOKS:

Applebaum D., Lévy Processes and Stochastic Calculus, 2nd Edition, Cambridge University Press, (2009).

Samorodnitsky G., Taqqu M., Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance, Chapman and all, New York, (1994).

Timetable (to be announced): 6 two-hour lectures.