

## PhD Course " Optimal control and applications"

Duration: 30 hours

Starting date: March 2021

Contact: for more information write to [falcone@mat.uniroma1.it](mailto:falcone@mat.uniroma1.it)

The course aims to be an introduction to optimal control theory for systems driven by ordinary differential equations and to present some recent applications to reinforcement learning and mean field games. We will discuss the main results related to the dynamic programming approach and the solution of the corresponding Hamilton-Jacobi equations giving some hints also on the numerical approximation of those problems.

The course is organized in three modules of about 10 hours each:  
Optimal control, Reinforcement learning, Mean field games.

### Tentative program

*Optimal control, M. Falcone (Sapienza)*

Introduction to some classical problems of deterministic control theory. The direct approach and Pontryagin principle. Dynamic programming and Hamilton-Jacobi-Bellman equations. Value function and viscosity solutions. Feedback reconstruction. Numerical approximation and algorithms. Optimal control of diffusion processes .

*Reinforcement learning, M. Palladino (GSSI, L'Aquila)*

Introduction to Reinforcement Learning (RL). Model free vs Model based RL. Bayesian RL. Connection between Optimal control and Reinforcement Learning. Modeling uncertainty in RL.

*Mean field games, F. Silva (Limoges)*

Nash equilibria in differential games with infinitely many players and Mean Field Games. Some applications in economics, finance, social sciences. Characterization of equilibria via a system of nonlinear PDEs of Hamilton-jacobi-Bellman and Fokker-Planck equations. Existence and uniqueness. Some hints on numerics.

### References

- M. Bardi and I. Capuzzo-Dolcetta, *Optimal Control and Viscosity Solutions of Hamilton- Jacobi-Bellman Equations*. Birkhauser, Basel, 1997
- P. Cardaliaguet, *Notes on Mean Field Games*, September 2013, <https://www.ceremade.dauphine.fr/~cardaliaguet/MFG20130420.pdf>
- E. Carlini, F.J.Silva, *A Fully Discrete Semi-Lagrangian Scheme for a First Order Mean Field Game Problem*, *SIAM Journal on Numerical Analysis* 52 (2014) no. 1, 45-67.
- M. P. Deisenroth, C. E. Rasmussen, "PILCO: A Model-Based and Data-Efficient approach to policy search", *Proceedings of the 28th International Conference on Machine Learning, ICML 2011*.
- M.Falcone, R.Ferretti, *Semi-Lagrangian Approximation Schemes for Linear and Hamilton-Jacobi equations*, *SIAM*, 2013.
- R. W. Murray, M. Palladino, "A Model for system uncertainty in Reinforcement Learning", *Systems and Control Letters*, 122, 2018, pg. 24-31.
- R. S. Sutton, A. G. Barto, "Reinforcement Learning, second edition: An Introduction", MIT press, 2018.